

# ALGEBRA

## QUADRATIC EQUATION PROBLEMS

- A1. Druby thinks of a number, squares it, then subtracts 3 times the number she started with and adds 4. She ends up with 22. What number did she start with?
- A2. A rectangle is 5m longer than it is wide. It has an area of  $36\text{m}^2$ . How wide is it?
- A3. Ann is 4 years older than John. The product of their ages is 60. How old is John?
- A4. Furry and Mothballs start with the same number. Furry squares it and subtracts 13. Mothballs multiplies it by 5 and adds 1. To their surprise, they end up with the same number. What number did they start with?
- A5. Aggro's pool is rectangular and 3m longer than it is wide. If its area is  $54\text{m}^2$ , what are the dimensions?
- A6. Reuben is 2 years older than Fred. The product of their ages is 48. How old are they?
- A7. The length of a Korean tennis court is 6m less than twice its width. If its area is  $80\text{m}^2$ , what are its dimensions?
- A8. Surley's dining table is rectangular. Its length is 3m more than twice its width. If its area is  $14\text{m}^2$ , what is its perimeter?
- A9. A rectangular garden bed is twice as long as it is wide. If its perimeter is 9m, what is its area?
- A10. A right-angle triangle has its shortest side 2m shorter than the next shortest side. If its area is  $24\text{m}^2$ , what is the length of the shortest side?
- A11. Another right-angle triangle has its hypotenuse 15cm long. The difference between the lengths of the other two sides is 3cm. What is its area?
- A12. Yet another right-angle triangle has its second longest side 17cm longer than its shortest side and an area of  $168\text{cm}^2$ . What is its perimeter?
- B1. A rectangle is 3m longer than it is wide. If its area is 20m, how wide is it?
- B2. The height of a shell  $t$  seconds after being fired is given by the formula  $h=60t-5t^2$ . How many seconds after firing will it be 66m high?
- B3. The base of a triangle is 4cm more than the perpendicular height. Its area is  $23\text{cm}^2$ . What is the base length?
- B4. Find two numbers that differ by 5 and have a product of 30.
- B5. The stopping distance,  $d$ , in metres for a car travelling at speed  $s$  in km/h is given by  $d=s+s^2/40$ . At what speed is the stopping distance 40m?

- B6. Some people hired a bus and agreed to share the cost of \$360. 4 people then pulled out and the rest had to pay an extra \$3 each to make up the shortfall. How many people went?
- B7. Patty had to travel 120km. She did the first 60km at constant speed, then the last 60km 5km/h faster. The whole journey took 8 hours. How fast did she go for the first 60km?
- B8. A boat can travel at 10km/h through the water. Travelling 40km with the current takes 40 minutes longer than travelling the same distance against the current. How fast is the current?
- B9. A 6m ladder rests against a wall with the top end 2m further up the wall than the bottom end is from the wall. How far from the wall is the bottom of the ladder?
- B10. A square lawn 10m by 10m has a concrete path around it of uniform width. The outside edge of the path is a square also. If the area of the path is  $50\text{m}^2$ , how wide is the path?
- B11. A cylinder has a height of 8cm and a surface area of  $130\text{cm}^2$ . What is its diameter?
- B12. Two concentric circles are drawn on a wall, the outer one having a radius 1m more than the inner one. The space between the two is  $15\text{m}^2$ . What is the radius of the smaller one?
- B13. Una walked 20km. If she had walked 1km/h faster she would have saved half an hour. How fast did she walk?
- B14. Sluggy spent \$45 a week on beer. When the price of a beer went up 5c he worked out that in order not to spend any more money, he would have to drink 3 less beers a week. What was the new price of a beer?

## RATIONAL POWER AND EXPONENTIAL EQUATION PROBLEMS

- A1. A number squared is 49. What is the number?
- A2. The square root of a number is 3. What is the number?
- A3. The square of a number is 196. What is the number?
- A4. The square of a number is 40. What is the number?
- A5. What number has a square root of 13.4?
- A6. The cube of a number is 100. What is the number?
- A7. Find  $5^{2.7}$ .
- A8. A number raised to the power of 3.5 is 12. What is the number?
- A9. Solve:  $t^{0.7} = 32$ .
- A10. Desmond put \$200 in the bank with compound interest. The amount he would have after  $n$  years if the interest rate was  $i\%$  is given by the formula  $A = 200(1 + i/100)^n$ . If after 4 years he had \$270, what was the interest rate?
- A11. In Q12, if Desmond had \$300 at the end of 3 years, what would the interest rate have been?
- A12. Herby put \$8000 in the bank with compound interest. 5 years later he had \$9250. What interest rate was he getting?
- A13. Sarah's car decreased in value by a certain percentage each year. The formula for its value after  $n$  years is  $V = C(1-d/100)^n$ , where  $V$  is the value,  $C$  is the initial cost,  $d$  is the percent annual depreciation and  $n$  is the age in years. If it cost her \$16000 and 5 years later was worth \$9000, by what percentage did its value drop each year?
- A14. Doreen spent \$2000 on a computer. 3 years later it was worth \$1100. What was the effective annual percentage depreciation?
- A15. Silas put \$1200 in the bank.  $4\frac{1}{2}$  years later he had \$1700. What rate of compound interest was he getting?
- A16. Gertrude spent \$400 on a silage pump.  $3\frac{1}{4}$  years later it was worth \$250. What was the effective annual depreciation?
- A17. Mortgage Morgan borrowed \$15 000 from Metway. 2 years 6 months later he paid back \$21 549 to pay off the loan. What was the compound interest rate?
- A18. Frilly bought some shares for \$3000. 4 years later she got \$4300 for them. What was her effective return on them? [Effective return means the rate of compound interest she would have to get on the money she invested to end up as well off at the end.]
- A19. Michael bought a Picasso for \$92 000. 2.7 years later he sold it for \$103 000. What effective return did he get on his investment?
- A20. The half-life of a radioactive isotope is the time taken for half of it to decay. If 75% of an isotope decays in 6 years, what is its half-life?
- A21. What is the half-life of a radioactive isotope if 90% of it decays in 6 hours?
- B1. If  $4^n$  is 64, what is  $n$ ?
- B2. To what power would you have to raise 3 to make 243?
- B3. To what power would you have to raise 2 to make 7?
- B4. Solve:  $6^n = 36$

- B5. Solve:  $12^n=90$
- B6. Solve:  $5^n=2$
- B7. Solve  $4.3^n=0.28$
- B8. To what power would you have to raise  $\frac{1}{2}$  to get 0.78?
- B9. Solve:  $0.7^s=5.2$
- B10. Durrall put \$1000 in the bank at 8% compound interest. The amount in the bank after  $n$  years is given by the formula  $A=P(1.08)^n$ . How long would it take him to end up with \$1500?
- B11. Mrs Crabtree invests \$2000 at 7% compound interest. How long will it take her to end up with \$2300?
- B12. Go-Anna put money in the bank at 12% compound. How long would it take for her money to double?
- B13. Each year the value of a car drops by 12%. Thus the formula for the value of a car  $n$  years old is  $V=C(.88)^n$ , where  $V$  is the value and  $C$  is the new cost. After how long would a car be worth 40% of its original value?
- B14. Each year the value of a machine drops by 15%. How long will it take for the value to halve?
- B15. The half-life of a radioactive isotope is 12 years. What fraction of it would be left after 24 years?
- B16. In Q15, what fraction would be left after 9 years?
- B17. Again in Q15, How long would it be before on tenth of it was left?
- B18. A bacterial colony grows steadily such that its size doubles every 24 hours. How long would it take 100 bacteria to grow to 10 000 bacteria?
- C1. Hasman put \$600 in the bank at 5% compound interest. How much would he have after 4 years 5 months?
- C2. In question C1, how long would Hasman have to leave his money in there to end up with \$6 000?
- C3. In question C1, what interest rate would Hasman have needed to get \$100 interest in the first 2 years?
- C4. Mrs Jones bought some antique furniture for \$2 700 and sold it 4 years later for \$9 250. What was the effective return on her investment? [See question A18.]
- C5. If Mrs Jones' furniture continued to appreciate at the same rate, how much longer would she have had to wait for it to be worth \$10 000?
- C6. A crowd of 500 people are caught in a paddock at the base of a cliff. A blind man is standing at the top of the cliff firing shots into the crowd one every 5 seconds. The probability of killing someone is proportional to the number of live people in the paddock. Thus the number of live people decreases exponentially with time and the formula for the number of live people  $h$  hours after he starts is  $n=500x.77^h$ . How many people will be alive after 6

hours?

- C7. In question C6, how long would it be before 200 people were dead?
- C8. In question C6, if the blind man had fired more often, would the base (the .77 in the example given) have been higher or lower?
- C9. In question C8, what would the base have been if it had taken him 5 hours to kill half the people?
- C10. The population of rabbits in a laboratory increases exponentially according to the formula  $p=ar^w$ , where  $a$  is the initial number,  $p$  is the number  $w$  weeks later and  $r$  is a constant. If  $r=1.12$ , how long would it take the population to double?
- C11. In question C10, what would  $r$  have to be to get the population to double in 12 weeks?
- C12. In a bacteria culture the number of bacteria doubles every 40 hours. How long would it take for the population to treble?
- C13. If the culture in question C12 started with 50 bacteria, write a formula for the number of bacteria  $h$  hours later.
- C14. Use your formula from question C13 to find how many bacteria there would be 1 week later.
- C15. Write a formula for the amount of money in the bank  $t$  years after depositing \$2 000 at 7% compound interest.
- C16. Use your formula from question 15 to find how long it would take before there was \$3 200 in the bank
- C17. 1000 people play a game of Russian Roulette. All players have a 1 in 6 chance of dying in each round. How many players would you expect to be alive after 7 rounds?
- C18. Write a formula for the number of players left alive after  $n$  rounds of the game in question C17.
- C19. Use your formula from question C18 to find out how many rounds it would most likely take for 9 tenths of the players to have received brain damage.
- C19. Solve:  $r\sqrt{r}=6$
- C20. Solve:  $a(3\sqrt{a})=20/a$
- C21. Solve for  $a$ :  $a^n \times \sqrt{a}=20$

### SIMULTANEOUS EQUATION PROBLEMS

- A1. Grubel thinks of 2 numbers. One is 4 more than the other. When added together, they make 16. What are the 2 numbers?
- A2. Axel thinks of two numbers. Their difference is 5 and their sum 31. What are they?

- A3. Rose and Harley are 3 years different in age. The sum of their ages is 37. How old are they?
- A4. Hungel's pool is rectangular. Its length is 6m greater than its width. Its perimeter is 28m. What are its dimensions?
- A5. Gargoil picked 3 lotto numbers. The second number was 7 less than the first; the last was 7 more than the first. The sum of the three was 45. What were the numbers?
- A6. Theodore drew a line, then another one 2cm longer, then another 2cm longer than that and so on until he had 6 lines. The total length was 1.2m. How long was each line?
- A7. Jodi's bank account had twice as much in it as Charlene's. Jodi then deposited another \$20. Between them they then had \$110. How much of that was Jodi's?
- A8. Mama had \$8 more than Pop who had 3 times as much as Grandpa. Between the three of them they had \$155. How much did Pop have?
- A9. Alf's weasel weighs 200g more than his 2 identical guinea pigs put together. His hamster weighs 40g less than one of his guinea pigs. The weasel, the hamster and one of the guinea pigs weigh 800g between them. How much does the weasel weigh by itself?
- A10. Tulie once had 5 rats but they all escaped before she could weigh them.
- A11. A rectangle is 20cm longer than it is wide. If its perimeter is 120cm, what is its area?
- A12. Fifi's rectangular pool is twice as long as it is wide. If its perimeter is 30m, what is its length?
- A13. Perdy painted a rectangular picture. Its length was 10cm more than twice its width. If its perimeter was 1.4m, what was its width?
- A14. Rasputin thought of a first number, then a second number 6 more than the first, then a third number 3 times the second, then a fourth number 20 less than the third. He then added them all up and got 24. What was the fourth number?
- A15. Lara had some money in the bank. Greta had \$25 more than Lara. Prudence had twice as much as Lara. Giles had 3 times as much as Greta plus another \$8. Between the four of them they had \$192. If Giles gave half his money to Greta, how much would she have?
- A16. Maisey has 2 more children than Flo, but only half as many as Dierdre who had 3 more than Bette. Between them all they had 7 kids. How many did Flo have?
- A17. Ergo worked for 6 days. Each day he got paid \$5 more than the previous day. His total wage for the 6 days was \$180. How much was he paid on the 4th day?
- A18. Lucy worked for 4 days and earned twice as much each day as on the previous day. She also got a \$100 bonus at the end, making her total pay \$278.50. How much did she earn on the third day?
- A19. Fegg earned \$34 a day. He then got a pay increase. Not satisfied with this he changed jobs to one that paid twice as much as his new pay. He worked one day at this rate then got another pay rise of \$4 a day. He then worked 3 more days at the higher rate, then got the sack. He earned a total of \$274 at the new job. How much did he earn on the first day there?
- A20. Kayleen made up some maths problems on Tuesday. On Wednesday she made up 3 times as many. On Thursday she made up 6 less than on Wednesday. On Friday she made up 7 more

than on Tuesday. If she made up 165 on the last 2 days, how many did she make up on the first 2 days?

- A21. Kayleen committed suicide on the weekend.
- B1. A table costs 3 times as much as a chair. 2 tables and 5 chairs cost \$275. What is the price of a) a table b) a chair
- B2. Fred and Ginger noticed that the sum of their ages was 68 years and the difference between them was 14 years. How old was a) Fred b) Ginger
- B3. 4 calculators and 2 text books cost \$122. 2 calculators and 5 text books cost \$129. Find the price of a) a calculator b) a textbook
- B4. The tortoise is  $1\frac{1}{2}$  times as old as the hare. The sum of their ages is 70. How old is a) the tortoise b) the hare
- B5. The carpenter weighs  $\frac{3}{8}$  as much as the walrus. The walrus weighs 80kg more than the carpenter. What is the weight of a) the carpenter b) the walrus
- B6. The road from Boulia to Mt Isa is 241km long. The gravel section is 37km shorter than the tar section. How long is the gravel section?
- B7. 2 loaves of bread and 5 fishes cost \$7.10. 5 loaves and 2 fishes cost \$6.20. Find the cost of 3 loaves and 3 fishes.
- B8. 5 large marbles and 12 small marbles weigh 196g. 5 small marbles weigh the same as 2 large marbles. Find the weight of 4 small marbles and 1 large marble.
- B9. 2 buses and 3 trucks can carry 149 passengers. A bus can carry 17 more passengers than a truck. How many passengers could be carried by 5 buses and 10 trucks?
- B10. The mean rainfall for April and May for Beenleigh is 186mm. The mean for May is 18mm less than for April. What is the mean for April?
- B11. 2 weasels and an owl eat 21 mice per day. 3 weasels and 4 owls eat 44 mice per day. How many would 1 weasel and 3 owls eat per day?
- B12. The length:width ratio of a rectangle is 3:5. If its perimeter is 30m, how long is it?
- B13. Entry to see the dead fish is \$4 for Adults and \$1.50 for children. One day 217 people paid a total of \$528. How many of them were children?
- B14. Peter and Wendy each think of a number. Peter's number multiplied by 3 is 4 more than Wendy's number multiplied by 4. The two numbers add to 13. What is the difference between them?
- B15. The area of a rectangle is  $14\text{cm}^2$ . Its perimeter is 20cm. Find its length.
- B16. A grocer buys enough stuff to fill 5 trucks and 3 vans or 3 trucks and 11 vans. What is the ratio of the capacities of trucks and vans?
- B17. A right angle triangle has area  $24\text{cm}^2$  and perimeter 19cm. How long is the hypotenuse?

- B18. Solve:  $2^{(x+y)}=50$ ;  $x-y=3$
- B19. Mr Taylor invested \$500 at 12% compound interest while he went to New Zealand. Then he took his money with the interest and deposited it again at the same interest rate while he went to Nigeria. When he got back from Nigeria, he had \$780. He spent 18 months longer in Nigeria than in New Zealand. How long did he spend in Nigeria?
- B20. Solve:  $(xy)^{3.5}=20$ ;  $\sqrt{(x+2y)}=2.1$
- C1. Karen is 3 years older than Josh. Josh is 5 times as old as Saliva. The sum of their ages is 25 years. How old is each?
- C2. A brick weighs twice as much as a small paver and 2kg less than a large paver. A large paver, a brick and a small paver together weigh 9.5kg. How much does each weigh?
- C3. At Frank's Pet Shop 2 parrots, 3 ferrets and a barracuda cost \$282. 4 parrots and 8 barracuda cost \$336. 5 ferrets and 1 parrot cost \$310. What would be the cost of one of each?
- C4. Sesame has 3 pets. The dog and the wombat together weigh 39kg more than the vulture. The dog and the vulture together weigh 9kg less than the wombat. The wombat weighs 4 times as much as the vulture. What is their combined weight?
- C5. Three families travel from Llywydd to Fyngogg on the train. The Griffiths have 2 adults and 5 children and pay \$50. The Evans have one adult, 3 children and a dog and they pay \$31. The Llewelyns have 3 adults, 11 children and 7 dogs and they pay \$117. The Diffyds have 2 adults, 1 child and no dogs. How much will they have to pay?

## 2-VARIABLE FORMULAE

1. Get some dotted paper with the dots in a square pattern. Make some polygons by connecting dots. For each polygon record the area, the number of dots on the perimeter and the number of dots inside. Find the relation between these three variables and write it algebraically with each of the 3 variables as the subject of the formula.
2. A company makes square pools with paved surrounds. The pavers are 50cm square. The dimensions of the pools are multiples of 50cm. One or more rows of pavers can surround the pool. Find a formula for  $p$ , the number of pavers needed in terms of  $s$ , the side length of the pool in metres and  $r$ , the number of rows of pavers required.